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On a Distribution of First Conception Delays in the Presence of Adolescent Sterility

Introduction

'T'HE probability models for the time of first conception are generally based on the assumption that all women under consideration are biologically mature enough to conceive. However, in countries like India, it is not uncommon to observe that the age at consummation of marriage and menarche coincide in a large number of cases, especially in rural areas. As all women will not be having reproductive maturity at the time of menarche, due consideration should be given to the period of adolescent sterility, which is the time interval between menarche and the attainment of full biological maturity to conceive, when probability models are drawn up for first conception delays. Apart from this, such details as the expected proportion of adolescent sterile women at various ages among those married are in their own right, of immense interest to demographers in arriving at reliable estimates of fertility.

Talwar (1965) found the extent of adolescent sterility among a group of Bengalee woman, by analysing the intervals between consummation of marriage aad first birth for a series of cohorts. Pathak and Prasad (1977) and Pathak (1978) derived probability models for the time of first conception with provision for adolescent sterility among a section of the women under consideration and evolved an indirect method to estimate the percentage of women who have not attained biological maturity at the time of marriage. The assumption in the above models, that the component of adolescent sterility continues to play indefinitely among those who are not biologically mature to conceive, was later modified by Saxena and Nair (1982). They presented a modified model in which the effect of adolescent sterility was assumed to cease after a specified time from marriage and used the model to obtain revised estimates of the extent of adolescent sterility and fecundability. Nair (1983) evolved a continuous stochastic model along similar lines, where each woman at any time after marriage is classified as belonging to one of the states of being (1) biologically immature to conceive but exposed to the risk of ovulation, (2) ovulating and exposed to the risk of conception and (3) conceived.

The aim of the present paper is to develop a distribution for first conception delays in the presence of adolescent sterility by treating time as discrete and to show that the model provides a close fit in a real situation. A noteworthy feature here is that besides obtaining estimates of the extent of adolescent sterility, fecundability etc. as in the earlier studies, the present investigation obtains explicit formulas that enable the calculation of the expected number of women in each of the above states and the corresponding probabilities. This extra output, enables the demographer to arrive at fertility estimates based on the expected number exposed to risk, an information which is otherwise not available in situations where the age at marriage is considerably low.

Distribution of First Conception Times

We assume that there is a group of K women who are all married at the same age (reckoned as time zero in the model) among whom $K(1 - \theta)$, $0 < \theta < 1$, are biologically mature enough to conceive and are exposed to the risk of conception and the remaining $K\theta$, yet to attain biological maturity and are exposed to the risk of ovulation. Denote by L_t, M_t, N_t the number of women in the states 'adolescent sterile', 'ovulating' and 'conceived' respectively at any time t. Further, letp, r respectively be the probabilities of a transition from the first state to the second and from the second state to the third. The number of women that changes from the first state in a unit of time (taken here as a month) from t, is solely due to the shift from that state to the following one at a rate of p so that we can write

$$L_t - L_{t+1} = pL_t. \qquad (1)$$

This gives the difference equation

$$L_{t+1} = qL_t, \quad q = 1 - p \qquad (2)$$

whose solution is

$$L_t = q^t L_0. \qquad (3)$$

Remembering that at $t = 0$, the number present in the first state is $K\theta$, equation (3) takes the form

$$L_t = K\theta\, q^t. \qquad (4)$$

Now, the change in state 2, obtained as the net result of transitions from the first state to the second and those from the second to third, is represented by the equation

$$M_t - M_{t+1} = -pL_t + rM_t$$

$$M_{t+1} = sM_t + K\theta\, p\, q^t, \quad s = 1 - r. \tag{5}$$

The solution of (5) is

$$M_t = \begin{cases} q^t M_0 + K\theta\,(1 - q^t) & q = s \\ s^t M_0 + \dfrac{K\theta p}{s - q}\,(s^t - q^t), & q \neq s \end{cases} \tag{6}$$

where $M_0 = (1 - \theta) K$, from the initial conditions.

For a woman chosen at random the probability that her waiting time to first conception T, exceeds t is given by

$$P(T > t) = \frac{L_t + M_t}{K}$$

$$= \begin{cases} \theta\, q^t + (1 - \theta)\, s^t + \dfrac{\theta p}{s - q}\,(s^t - q^t) & q \neq s \\ \theta + (1 - \theta)\, q^t & q = s \end{cases} \tag{7}$$

When $q \neq s$, equation (7) can be put as

$$P(T > t) = \alpha\, q^t + (1 - \alpha)\, s^t \tag{8}$$

where

$$\alpha = \left(1 - \frac{p}{s - q}\right)\theta = \frac{r\theta}{r - p} \tag{9}$$

From equation (8), the distribution of T is obtained. This is

$$f(t) = P(T = t)$$

$$= P(T > t - 1) - P(T > t)$$

$$= \alpha\, pq^{t-1} + (1 - \alpha)\, r\, s^{t-1}, \quad t = 1, 2, \ldots \tag{10}$$

Notice that the usual geometric law for first conception times appears as a special case of (10) when $\alpha = 0$, which in turn implies $\theta = 0$ by virtue of (9); that is, when adolescent sterility is absent.

Estimation of Parameters

In order to estimate the parameters θ, p and r, we obtain the first three factorial moments of (10), with the corresponding observed factorial moments $m_{(1)}$, $m_{(2)}$ and $m_{(3)}$. This leads to the simultaneous equations

$$\alpha x + (1 - \alpha)\, y = a_1 \tag{11}$$

$$\alpha x\,(x - 1) + (1 - \alpha)\, y\,(y - 1) = a_2 \tag{12}$$

$$\alpha x\,(x - 1)^2 + (1 - \alpha)\, y\,(y - 1)^2 = a_3 \tag{13}$$

where $a_1 = m_{(1)}$, $a_2 = \frac{1}{2}\, m_{(2)}$, $a_3 = (1/6)\, m_{(3)}$, $x = 1/p$, $y = 1/r$.

From equation (11), it is easy to see that

$$\alpha = \frac{a_1 - y}{x - y} \tag{14}$$

Rewriting (12) as

$$\alpha\,(x - y)\,(x + y - 1) + y^2 - y = a_2$$

and then inserting into it, the value $a_1 - y$ of $\alpha\,(x - y)$, we get after simplification the relationship

$$y = \frac{a_1 + a_2 - a_1 x}{a_1 - x}. \tag{15}$$

From equations (11) and (12),

$$a_2 - a_1\,(x - 1) = (1 - \alpha)\, y\,(y - x) \tag{16}$$

and from (12) and (13),

$$a_3 - a_2\,(x - 1) = (1 - \alpha)\, y\,(y - 1)\,(y - x). \tag{17}$$

Eliminating α, between (16) and (17),

$$a_3 - a_2\,(x - 1) = (y - 1)\,(a_2 - a_1 x + a_1). \tag{18}$$

Finally, using (15) in (18) leaves the quadratic equation

$$(a_1 + a_2 - a_1^2)\, x^2 + (a_1^2 + a_1 a_2 - a_1 - 2a_2 - a_3)\, x + (a_1 a_3 - a_2^2) = 0$$

which can be solved for x. Using this value in (14) and (15) α and y and hence p, r and θ can be obtained. This completes the procedure of estimation.

Application

As an illustration of how the model works in a practical situation, it was applied to the data reproduced in Pathak (1978), on the times of first concep tion of a cohort of 239 women married at the age of 15 years. With the values $a_1 = 38.7$, $a_2 = 1700.75$ and $a_3 = 83604.1$, the estimates of θ, p and r were obtained to be

$$\hat{\theta} = 0.2092 \quad \hat{p} = 0.0171 \quad \hat{r} = 0.0378$$

The expected frequencies based on these estimates, along with the obsened frequencies presented in Table 1, show clearly that the model fits the data quite closely.

TABLE 1-OBSERVED AND EXPECTED NUMBER OF CONCEPTIONS

Time (in months)	*Observed Frequency*	*Expected Frequency*	
1-7	47	45.28	
8-31	96	95.46	
32-55	50	45.13	
56-79	17	22.80	$X^2 = 2.957$
80-103	11	12.08	
104-127	5	6.92	d.f. = 4
128-152	5	4.14	
152 +	8	7.24	

The values of the parameters when substituted in equation (4) and (6), provide the expected number of women who are respectively in the adolescent sterile and ovulating states at various time points. Also since the number of women in the three states at any time should add upto the initial cohort size, knowledge of the numbers in the first two states automatically determines those in the third. The expected percentage of women in each category at yearly intervals after marriage are given in Table 2.

TABLE 2—PERCENTAGE OF WOMEN IN EACH STATE

Time (in months)	*Expected Percentage of Women who are*		
	Adolescent	*Ovulating*	*Conceived*
0	20.92	79.08	..
12	17.15	53.14	29.71
24	13.81	35.98	50.21
36	11.30	24.69	64.01
48	9.21	17.15	73.64
60	7.53	12.13	80.34
72	5.86	8.79	85.35
84	4.60	6.69	88.71
96	4.18	5.02	90.80

The probability for a regularly ovulating woman to conceive in a year is given by 0.37. Hence the mean time required for conception works out to be approximately 2.7 years for such women. On the other hand, in the presence of adolescent sterility, the average time taken by a woman is nearly 3.2 years. The difference in these two averages provides sufficient indication of the force of adolescent sterility on conception delays. Further from Table 2, it can be seen that the median age at which the incidence of adolescent sterility cease to operate is around 20 years. Thus the model appears to elicit more information than the others existing in literature and further the fact that it needs as input only the times to first conception which can be readily obtained for any population, makes the model widely applicable.

Acknowledgement

I am greatly indebted to Dr. R. Ramakumar, Professor of Demography, University of Kerala for his valuable guidance in the preparation of this paper and to Dr. P. C. Saxena, I. I. P. S., Bombay who directed my interest to this topic.

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